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ABSTRACT

In this work a new type of partially polarized and partially coherent sources is proposed. The coherence characteristics of these sources are dependent on the difference of the radial distances from the source center of the two points to be compared. The coherence is perfect for points located on the same circle centered on the source center and decreases for points that belongs to different concentric circles. The maximum attainable coherence is related to the degree of polarization of the source. Coherence and polarization characteristics of this kind of fields at the source plane and upon free space propagation are analyzed in detail for a simple case. For the particular presented example, a partially polarized and partially coherent field is obtained, whose polarization properties are invariant in propagation.

Keywords: Coherence, Polarization, Beam propagation

1. INTRODUCTION

There is an increasing interest in finding physically realizable partially coherent sources with coherence characteristics different from those of the usual Schell model type.^{1–14} It is also expected that interesting results could be found if the vectorial nature of light is taken into account. The coherence and polarization characteristics of a light source can be appropriately described by its cross-spectral density matrix (CSDM).^{15,16} Recent findings of the coherency theory, such as the superposition rule and its generalization to the vectorial case,^{1–3,17} have eased the study of new realizable CSDM's.

In this work a new type of sources is proposed, which are both partially polarized and partially coherent. Their coherence characteristics are not shift invariant, as it occurs for sources of the Schell-model type, but depend on the difference between the moduli of the two point vectors considered.¹³ We shall call them polarized pseudo-Schell model sources. The coherence and polarization characteristics of this kind of fields at the plane source and upon free space propagation will be analyzed in detail.

2. KEY DEFINITIONS

The cross-spectral density matrix $\hat{G}(\mathbf{r}_1, \mathbf{r}_2)$ is an appropriate tool for describing stochastic, statistically stationary up to second order, electromagnetic light sources.^{15,16} Under paraxial approximation, the CSDM is a 2×2 matrix that gives the correlations between different field components at two arbitrary positions across the source plane. The two points are denoted by the vectors $\mathbf{r}_j = (r_j, \theta_j)$ with $j = 1, 2$ where (r_j, θ_j) are the radial and angular coordinates, respectively.

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In the scalar treatment, a genuine cross-spectral density must satisfy the nonnegativeness condition.¹⁵ However, to check directly this nonnegativeness condition for a given cross-spectral density is not an easy task. Some years ago a necessary and sufficient condition that guarantees such nonnegativeness has been established.^{3,17} This condition gives a rule, the so-called superposition rule,¹⁷ to easily construct physically realizable cross-spectral densities. An extension of this rule for the electromagnetic case has been derived,^{1,2} so it can be easily checked if a proposed matrix represents a genuine CSDM.

The coherence properties of an electromagnetic source can be studied through the electromagnetic degree of coherence^{18–20}

$$\mu_{STF}^2(\mathbf{r}_1, \mathbf{r}_2, z) = \frac{\text{tr}\{\hat{G}(\mathbf{r}_1, \mathbf{r}_2, z) \hat{G}^\dagger(\mathbf{r}_1, \mathbf{r}_2, z)\}}{\text{tr}\{\hat{G}(\mathbf{r}_1, \mathbf{r}_1, z)\} \text{tr}\{\hat{G}(\mathbf{r}_2, \mathbf{r}_2, z)\}}, \quad (1)$$

where “tr” and \dagger denote the trace and the transpose conjugate of a matrix, respectively.

The evaluation of the CSDM at the same point $\mathbf{r} = \mathbf{r}_1 = \mathbf{r}_2$ results in the polarization matrix¹⁵ that describes all the polarization characteristics of the field at any point in its cross section. On the other hand, the polarization properties of light beams are usually studied through a set of four parameters usually arranged in a vector, the so-called Stokes vector $\mathbf{S} = (S_0, S_1, S_2, S_3)^T$,^{21,22} where superscript T denotes transpose. In general, \mathbf{S} is a point-dependent vector that can be derived from the polarization matrix as¹⁵

$$S_i(\mathbf{r}, z) = \text{tr}\{\hat{G}(\mathbf{r}, \mathbf{r}, z) \hat{\sigma}_i\}, \quad (2)$$

where $\hat{\sigma}_i$, ($i = 0, 1, 2, 3$), are the 2×2 identity matrix together with the three Pauli matrices¹⁵

$$\hat{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \hat{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \hat{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (3)$$

The Stokes vector gives the same information as that contained in the polarization matrix, but the components of the former are measurable quantities. For example, $S_0(\mathbf{r}, z) = \text{tr}\{\hat{G}(\mathbf{r}, \mathbf{r}, z)\}$ is the source intensity, and the remaining three represent the source intensity measured with suitable wave plates and linear polarizers placed in front of the detector.

The degree of polarization, P , which is the ratio of the polarized part of the field to the total irradiance,¹⁵ gives important information about the polarization of the field in a single parameter. It can be obtained at each point of the cross section from the polarization matrix as

$$P(\mathbf{r}, z) = \sqrt{1 - \frac{4 \det\{\hat{G}(\mathbf{r}, \mathbf{r}, z)\}}{(\text{tr}\{\hat{G}(\mathbf{r}, \mathbf{r}, z)\})^2}}, \quad (4)$$

where “det” stands for determinant.

To study the polarization and coherence characteristics of the field upon free space propagation, we assume paraxial approximation and propagation direction mainly along the z axis of a suitable reference frame. Then, the Fresnel diffraction integral¹⁵ allows us to obtain the CSDM at any plane $z = \text{const}$. From the knowledge of the CSDM at the source plane ($z = 0$), it turns out that¹⁵

$$\hat{G}(\mathbf{R}_1, \mathbf{R}_2, z) = \frac{1}{\lambda^2 z^2} \iint \hat{G}(\mathbf{r}_1, \mathbf{r}_2, 0) \exp\left[-\frac{i}{2z} (|\mathbf{R}_1 - \mathbf{r}_1|^2 - |\mathbf{R}_2 - \mathbf{r}_2|^2)\right] d^2 r_1 d^2 r_2, \quad (5)$$

where $\mathbf{R}_j = (R_j, \phi_j)$, with $j = 1, 2$, are two arbitrary position vectors across the plane $z = \text{const}$.

3. SOURCE MODEL

Let us consider a source at the plane $z = 0$ described by a matrix of the form

$$\hat{G}(\mathbf{r}_1, \mathbf{r}_2, 0) = \tau(\mathbf{r}_1)^* \tau(\mathbf{r}_2) \int_0^\infty \int_0^{2\pi} \hat{A}^\dagger(\mathbf{r}_1, \boldsymbol{\sigma}) \hat{A}(\mathbf{r}_2, \boldsymbol{\sigma}) \eta d\eta d\alpha, \quad (6)$$

where $\mathbf{r}_j = (r_j, \theta_j)$ with $j = 1, 2$ are two arbitrary positions across the source plane, $\boldsymbol{\sigma} = (\eta, \alpha)$, $\tau(\mathbf{r})$ is a complex function, and

$$\hat{A}(\mathbf{r}_1, \boldsymbol{\sigma}) = \hat{H}(\eta) \exp(ikr\eta \cos \alpha), \quad (7)$$

being $\hat{H}(\eta)$ an arbitrary 2×2 matrix, i the imaginary unit, and k the wave number. Due to the way in which $\hat{G}(\mathbf{r}_1, \mathbf{r}_2)$ has been constructed, it represents a genuine cross spectral density matrix.¹⁻³

Substitution from Eq. (7) into Eq. (6) gives

$$\hat{G}(\mathbf{r}_1, \mathbf{r}_2, 0) = \tau(\mathbf{r}_1)^* \tau(\mathbf{r}_2) \hat{G}_0(r_2 - r_1), \quad (8)$$

where

$$\hat{G}_0(r_2 - r_1) = 2\pi \int_0^\infty \hat{H}^\dagger(\eta) \hat{H}(\eta) J_0[k\eta(r_2 - r_1)] \eta d\eta, \quad (9)$$

and $J_\nu(\cdot)$ is the Bessel function of first kind and order ν .²³ It can be noted that the intensity of the source, except for a constant factor, is determined by the function $\tau(\mathbf{r})$, while its polarization and coherence properties are determined by the matrix $\hat{G}_0(r_2 - r_1)$.

To illustrate the properties of this kind of sources, we consider the following particular choice for $\hat{H}(\eta)$:

$$\hat{H}(\eta) = \begin{pmatrix} h_{xx} & 0 \\ 0 & h_{yy} \end{pmatrix} \text{circ}\left(\frac{\eta}{b}\right), \quad (10)$$

being h_{xx} and h_{yy} two arbitrary dimensionless constant coefficients, $\text{circ}(t) = 1$ for $|t| \leq 1$ and 0 otherwise, and b a parameter related to the area where the coherence of the source is significantly high.

For this choice of $\hat{H}(\eta)$ we obtain

$$\hat{G}_0(r_2 - r_1) = \begin{pmatrix} |h_{xx}|^2 & 0 \\ 0 & |h_{yy}|^2 \end{pmatrix} \pi b^2 \frac{2J_1[bk(r_2 - r_1)]}{bk(r_2 - r_1)}, \quad (11)$$

and the squared electromagnetic degree of coherence across the source turns out to be

$$\mu_{STF}^2(z=0) = \frac{|h_{xx}|^4 + |h_{yy}|^4}{(|h_{xx}|^2 + |h_{yy}|^2)^2} \left[\frac{2J_1[bk(r_2 - r_1)]}{bk(r_2 - r_1)} \right]^2. \quad (12)$$

It reaches its maximum value, $\mu_{STF,Max}^2(z=0)$, equal to

$$\mu_{STF,Max}^2(z=0) = \frac{|h_{xx}|^4 + |h_{yy}|^4}{(|h_{xx}|^2 + |h_{yy}|^2)^2}, \quad (13)$$

when $r_2 = r_1$, that is, for concentric rings around the source center. From this maximum value, the electromagnetic degree of coherence decreases and presents an oscillatory behavior for increasing values of $|r_2 - r_1|$. The distance $|r_2 - r_1|$ for which the first null of the electromagnetic degree of coherence is reached depends on the single parameter b .

On the other hand, the polarization properties are determined by the following Stokes vector

$$\mathbf{S}(\mathbf{r}, z=0) = |\tau(\mathbf{r})|^2 \pi b^2 \begin{pmatrix} |h_{xx}|^2 + |h_{yy}|^2 \\ |h_{xx}|^2 - |h_{yy}|^2 \\ 0 \\ 0 \end{pmatrix}, \quad (14)$$

that results in a uniformly partially polarized field with degree of polarization given by

$$P(z=0) = \frac{||h_{xx}|^2 - |h_{yy}|^2|}{|h_{xx}|^2 + |h_{yy}|^2}. \quad (15)$$

It can be noted that the degree of polarization can vary from zero, for $|h_{xx}|^2 = |h_{yy}|^2$, to one, when $|h_{yy}|^2 = 0$ or $|h_{xx}|^2 = 0$. Then, it is possible to generate unpolarized or totally polarized fields by varying the values of h_{xx} and h_{yy} . For different values of the coefficients h_{xx} and h_{yy} , the field is partially polarized.

Comparing Eqs. (13) and (15), it can be noted that the maximum attainable electromagnetic degree of coherence is related to the degree of polarization of the source in the following way

$$\mu_{STF,Max}^2(z=0) = \frac{1 + P^2(z=0)}{2}. \quad (16)$$

This means that, for totally polarized sources of this kind, the field is completely coherent ($\mu_{STF}^2 = 1$) for all points on a ring concentric to the source center $r_2 = r_1$. For the case of partially polarized sources belonging to the proposed class, the field is partially coherent even for $r_2 = r_1$. For the limiting case of unpolarized source, the maximum attainable electromagnetic degree of coherence is $1/2$.

4. FREE SPACE PROPAGATION

For the sources described in the previous section, the properties of field generated under free space propagation can be obtained assuming paraxial approximation. If we consider a source with $\tau(\mathbf{r}) = \tau(r) \exp(im\theta)$, the CSDM at a given z plane can be obtained by substituting Eq. (11) into Eq. (8) and the latter into Eq. (5). This gives

$$\begin{aligned} \hat{G}(\mathbf{R}_1, \mathbf{R}_2, z) &= \frac{4\pi^2}{\lambda^2 z^2} \exp\left[\frac{ik}{2z}(R_2^2 - R_1^2)\right] \exp[im(\phi_2 - \phi_1)] \begin{pmatrix} |h_{xx}|^2 & 0 \\ 0 & |h_{yy}|^2 \end{pmatrix} \frac{b^2}{2} \\ &\times \int_0^\infty \int_0^\infty \tau(r_1)^* \tau(r_2) \exp\left[\frac{ik}{2z}(r_2^2 - r_1^2)\right] \frac{2J_1(kb(r_2 - r_1))}{kb(r_2 - r_1)} J_m\left(\frac{k}{z}R_1r_1\right) J_m\left(\frac{k}{z}R_2r_2\right) r_1 r_2 dr_1 dr_2. \end{aligned} \quad (17)$$

In general, the coherence characteristics of the field affect the polarization, and the degree of polarization usually changes during propagation.²⁴⁻²⁸ We propose a class of partially coherent field for which the polarization properties remains invariant upon free-space propagation. On the other hand, the intensity profile and the electromagnetic degree of coherence of these sources vary with propagation.

5. SUMMARY

In the present work, a new type of electromagnetic sources has been proposed. They are partially polarized and partially coherent. For the simple example developed in this work, the polarization properties of the source remain invariant in propagation. The degree of polarization modulates the electromagnetic degree of coherence, which presents an annular shape across the source plane: it reaches its maximum value for points at equal distance from the center of the source and varies in the form given in Eq. (12) for other points.

ACKNOWLEDGMENTS

Spanish Ministerio de Economía y Competitividad project FIS2016-75147.

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